

**Final Exam**

**All of your answers/solutions/computations must be written on the provided Answer Sheet.**

**Vocabulary**

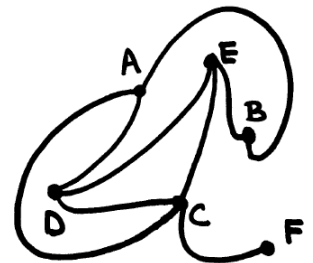
Match the following vocabulary terms with the best description.

- |                     |                                                                                                  |
|---------------------|--------------------------------------------------------------------------------------------------|
| 1. relatively prime | <b>A.</b> a cipher system that replaces the letters in a message with other characters uses this |
| 2. bridge           | <b>B.</b> a graph where every vertex is adjacent to every other vertex                           |
| 3. connected        | <b>C.</b> an edge that begins and ends at the same vertex                                        |
| 4. transposition    | <b>D.</b> a graph where a path exists from every vertex to every other vertex                    |
| 5. complete         | <b>E.</b> numbers greater than 1 that are only divisible by themselves and the number 1          |
|                     | <b>F.</b> a graph where the sum of all of the degrees of the vertices is even                    |
|                     | <b>G.</b> a graph whose edges intersect only at vertices                                         |
|                     | <b>H.</b> a cipher system that scrambles up the ordering of the letters in a message uses this   |
|                     | <b>J.</b> an edge that when removed will cause the graph to become disconnected                  |
|                     | <b>K.</b> a pair of numbers whose greatest common factor is 1                                    |

**Computational/Procedural**

Read each question and then select the most correct response or responses.

6. The number 86 is congruent to the number 50 in a modular 7 arithmetic system.  
**A.** True                      **B.** False                      **C.** Depends                      **D.** Impossible to tell
7. The graph shown at right does not contain an Euler path. Which of the following edges, when added to the graph, will result in the graph having an Euler path?  
**A.** CB                      **B.** BF                      **C.** AE                      **D.** BD
8. Which of the following is not a triangular number?  
**A.** 6                      **B.** 24                      **C.** 15                      **D.** 36
9. Which of the following are characteristics of an effective cipher system:  
**A.** the enciphering process must be kept secret to be effective  
**B.** it must use both transposition and poly-graphic substitution  
**C.** the enciphering process must be reversible  
**D.** it must be complex to the point that it will be impossible for enemy to break  
**E.** enciphering/deciphering can be accomplished relatively quickly



Questions #10 through #12 refer to the spreadsheet shown at right.

10. What would need to go into the question mark spot if you wanted to get the 2000 Profits to appear?

`=vlookup(12,$B$5:$G$15, -?- )`

- A.** 1    **B.** 2    **C.** 3    **D.** 4    **E.** E

11. What should appear if you type the following:

`=mod(G8+D14,$B7)`

- A.** 1    **B.** 2    **C.** 3    **D.** 4    **E.** 5

	A	B	C	D	E	F	G
3							
4		Store #	Location	Storage Capacity	2000 Profits	Open Sundays	Area Code
5		2	Los Angeles	5.0	\$3.4	Yes	213
6		3	Cupertino	2.0	\$2.1	Yes	408
7		5	Sacramento	3.1	\$4.2	Yes	916
8		8	Monterey	1.5	\$3.2	No	831
9		11	Fresno	2.5	\$3.8	No	559
10		12	San Diego	4.0	\$4.8	Yes	619
11		13	San Francisco	3.0	\$4.5	Yes	415
12		18	Modesto	1.0	\$2.3	No	209
13		25	San Jose	3.5	\$4.1	No	408
14		30	Oakland	6.0	\$6.2	Yes	510
15		31	Santa Barbara	0.5	\$3.9	No	805
16							

12. What would be displayed if you typed the following into a cell of the spreadsheet `=mid(C12,B6,4)`

13. Draw a single graph that exhibits all of the following characteristics:
- is not planar
  - has exactly 5 vertices
  - has exactly 12 edges
  - has exactly 3 loops

Answer the following being sure to show the work you did to arrive at your answer whenever possible.

14. Explain why a graph with 112 edges and 14 vertices cannot be a complete graph.
15. Using only addition & multiplication, solve  $27 + 3x \equiv 8 + 9x \pmod{5}$
16. Griddle's Cipher System combines an additive cipher in mod 26 with backwards writing. Decode FTQX which is a word that was enciphered using key 19. Be sure to clearly show each of the steps you take in the deciphering process.
17. Kevin decided to simplify his fashion life by devising a systematic way for choosing his daily attire. Starting January 1<sup>st</sup> of each year, he will wear the items listed first in the chart at right. On each subsequent day, he will wear the next item at the list. When he gets to the end of a particular list, he will move back to the top of that list. Use modular arithmetic to predict Kevin's attire on the 4<sup>th</sup> of July.

Pants/Shorts	Shirt	Shoes
Cargo Pants	Red Polo Shirt	Doc Martins
Acid-Washed Jeans	Brown T-Shirt	Sandals
Khaki Pants	Green Dress Shirt	Skechers
Paratrooper Shorts	Blue Thermal Shirt	
	Gray Bellarmine T-Shirt	
	Black Sweatshirt	

Jan	Feb	Mar	Apr	May	Jun
31	28	31	30	31	30

### Conceptual/Analysis

Answer the following using complete grammatically correct sentences.

18. Prime numbers are important enough that people today spent time trying to find the next new prime numbers. To do this, people employ the use of algorithms. Identify and explain two or three of the simpler algorithms used to test whether a number is prime and discuss how/why prime numbers are important in cryptography.
19. Discuss why poly-alphabetic ciphers are considered more secure than mono-alphabetic ciphers.
20. Read the following passage:

What is mathematics? If you ask this question of the first person you meet on the street, you will most likely hear that "mathematics is the study of number." If you insist that your respondent be more specific, you may elicit the suggestion that mathematics is "the science of numbers." . . . How does today's mathematician answer the question, "What is mathematics?" The most common answer is that mathematics is the science of patterns.

—An excerpt from *The Math Gene* by Keith Devlin

After reflecting on this passage and your experience this semester in this class, write a coherent essay addressing the following questions:

- \* How does your view of mathematics compare with the idea expressed in the above passage?
- \* How does your view of mathematics compare with the ideas presented in the movie Pi?
- \* Has your view of mathematics changed over the course of this semester? Explain why or why not.

### BONUS

Two burglars, Bob and Al, are captured near the scene of a burglary and are given the "third degree" separately by the police. Each has to choose whether or not to confess and implicate the other. If neither man confesses, then both will serve one year on a charge of carrying a concealed weapon. If each confesses and implicates the other, both will go to prison for 10 years. However, if one burglar confesses and implicates the other, and the other burglar does not confess, the one who has collaborated with the police will go free, while the other burglar will go to prison for 20 years on the maximum charge.

		Al	
		confess	don't
Bob	confess	10,10	0,20
	don't	20,0	1,1

- (A) Explain why this is not a zero-sum game.
- (B) Is there a Nash Equilibrium in this game? Explain.