BCP / AMDG
Discrete Math

Name: ______________________________

Fall Semester

Class Notes + Assignment

Introduction to Modular Arithmetic

Unlike the traditional arithmetic system where the integers are limitless, a ***modular arithmetic system*** *consists of a repeating cycle of a finite set of integers. For example, the number line in the traditional arithmetic system extends in both directions with integers increasing to the right and decreasing to the left while the number line in a modular arithmetic system repeats itself in both directions. To see how they compare, consider the following graphic:*

Modular Arithmetic Number Line

4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Traditional Arithmetic Number Line

By seeing these two number lines presented together, you can start to make some connections to the earlier activities, namely that a cycle length of 6 can be easily projected onto the traditional numbering system. In other words, it isn't difficult to predict what modular number (1, 2, 3, 4, 5, or 6) will correspond to (appear above) 317.

Another way to visualize this correspondence between the modular arithmetic and traditional arithmetic number lines would be to have the numbers "wrap around" from bottom to top as in this graphic:

...	-17	-11	-5	1	7	13	19	25	31	37	43	49	...
...	-16	-10	-4	2	8	14	20	26	32	38	44	50	...
...	-15	-9	-3	3	9	15	21	27	33	39	45	51	...
...	-14	-8	-2	4	10	16	22	28	34	40	46	52	...
...	-13	-7	-1	5	11	17	23	29	35	41	47	53	...
...	-12	-6	0	6	12	18	24	30	36	42	48	54	...

Some Formal Notation & Conventions

*** **WARNING** ***

Subtraction & Negative Integers are DANGEROUS!

It's Best to Avoid These

Complete the following on a separate sheet of loose-leaf paper and show all steps involved.

1) $29+14 \equiv x \pmod{6}$

2) $32+18 \equiv x \pmod{5}$

3) $40+x \equiv 15 \pmod{12}$

4) $x+24 \equiv 5 \pmod{7}$

5) $22-x \equiv 31 \pmod{8}$

6) $x-13 \equiv 22 \pmod{9}$

7) $31+x \equiv 9 \pmod{4}$

8) $2x+11 \equiv 8 \pmod{5}$

BCP / AMDG
Discrete Math

Name: ______________________________

Fall Semester

Class Notes + Assignment

Inverse Operations in Modular Arithmetic

It's time to formally investigate how addition and multiplication work in a modular arithmetic system. It will help to remember this graphic showing the correspondence between the modular and traditional arithmetic number lines.

...	-20	-13	-6	1	8	15	22	29	36	43	50	57	...
...	-19	-12	-5	2	9	16	23	30	37	44	51	58	...
...	-18	-11	-4	3	10	17	24	31	38	45	52	59	...
...	-17	-10	-3	4	11	18	25	32	39	46	53	60	...
...	-16	-9	-2	5	12	19	26	33	40	47	54	61	...
...	-15	-8	-1	6	13	20	27	34	41	48	55	62	...
...	-14	-7	0	7	14	21	28	35	42	49	56	63	...

Addition & Additive Inverses

Multiplication & Multiplicative Inverses

x	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

Complete the following on a separate sheet of loose-leaf paper and show all steps involved.

1) $17 + 3x \equiv 9 \pmod{7}$
2) $x - 18 \equiv 5 \pmod{7}$
3) $22 + 6x \equiv 2 \pmod{7}$
4) $32 - 2x \equiv 1 \pmod{7}$
5) $4x + 10 \equiv 2 \pmod{7}$
6) $5x + 11 \equiv 8 \pmod{7}$
7) $3x + 4 \equiv 12 \pmod{5}$
8) $5x + 8 \equiv 2 \pmod{6}$
9) $7x + 5 \equiv 2 \pmod{11}$